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by Dr. David S. Hart, of Stonington, Conn. We wrote to Dr. Hart upon the subject and he sent us a copy of his solution, which is brief, and, we think, satisfactory. About the same time we received the copy of Dr. H.'s solution, we received from Dr. Matteson, with a request to publish it, the foregoing solution of an analogous, but somewhat more general problem. We had intended to publish both solutions in the same number, but, at present, our space will not permit. We hope to find room for Dr. Hart's solution in a future No.—Ed.]

SOLUTION OF A PROBLEM.

BY PROF. JOSEPH FICKLIN, COLUMBIA, MISSOURI.

Problem.—Pairs of tangents to any conic section intersect at a given angle; determine the locus of their intersection.

Denote by (x, y) the co-ordinates of a point of intersection, and by $(x'\, y')$ a point of contact. The general equation of any conic section, when referred to the principal diameter and tangent through its vertex, is $y^2 = mx + nx^2$; therefore at the point (x', y') $y'^2 = mx' + nx'^2$; . . . . (1)
and for the tangent a of the angle formd by the axis of x, and a straight line through the points (x, y), and (x', y'), we have $a = (y - y') \div (x - x')$;
whence

$$y' = y - ax + ax'. \quad . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

Substituting this value of y' in (1) we obtain

$$(a^2 - n)x'^2 + (2ay - 2a^2x - m)x' + y^2 - 2axy + a^2x^2 = 0,$$

or

$$x'^2 + \frac{2ay - 2a^2x - m}{a^2 - n}x' = -\frac{y^2 - 2axy + a^2x^2}{a^2 - n} \quad . \; . \; . \; . \; . \; . \quad (3)$$

This equation gives two values for x'; but, since, by the conditions of the problem, the line through the points (x, y), (y', y') must have only one point, viz., $(x'\, y')$, in common with the curve, the two values of x', in (3), must be equal; ∴ $4(a^2 - n)(y^2 - 2axy\; a^2x^2) = (2ay - 2a^2x - m)^2$. Reducing this we have

$$a^2 - \frac{my + 2nxy}{mx + nx^2}a + \frac{m^2 + 4ny^2}{4(mx + nx^2)} = 0. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

The two values of a in (4) belong to the two tangents drawn from the points (x, y). Denoting these two values by a', a'', and the tangent of the angle which the two tangent lines make with each other by t, we get

$$t = \frac{a' - a''}{1 + a'a''}. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

But, by the theory of equations, we get from (4)

$$a'a''=\frac{m^2+4ny^2}{4(mx+nx^2)}, \text{ and } a'+a''=\frac{my+2nxy}{mx+nx^2};\ \therefore\ a'^2+2a'a''+a''^2$$

$$=\left(\frac{my+2nxy}{mx+nx^2}\right)^2, \text{ and } (a'-a'')^2=\left(\frac{my+2nxy}{mx+nx^2}\right)^2-\frac{m^2+4ny^2}{mx+nx^2}$$

$$=\frac{m^2y^2-m^3x-m^2nx^2}{(mx+nx^2)^2};$$

$$\therefore\quad t^2=\frac{m^2y^2-m^3x-m^2nx^2}{(mx+nx^2)^2}\div\left[1+\frac{m^2+4ny^2}{4(mx+nx^2)}\right]$$

$$=\frac{m^2y^2-m^3x-m^2nx^2}{(mx+nx^2)^2}\div\left[\frac{4mx+4nx^2+m^2+4ny^2}{4(mx+nx^2)}\right]$$

$$=\frac{m^2(y^2-mx-nx^2)}{(mx+nx^2)^2}\div\frac{(4mx+4nx^2+m^2+4ny^2)^2}{16(mx+nx^2)^2}$$

$$=\frac{16m^2(y^2-mx-nx^2)}{(4mx+4nx^2+m^2+4ny^2)^2}\ \ldots\ldots\ldots\ldots\ (6)$$

the equation of the locus required.

If $t=\tan 90^\circ=\infty$, then $4mx+4ny^2+m^2=0$ . . . . (7) the equation of a circle. If $n=0$,

$$t^2=\frac{16m^2(y^2-mx)}{(4mx+m^2)^2}=\frac{16(y^2-mx)}{(4x+m)^2},\ \ldots\ldots\ (8)$$

the equation of an hyperbola. If $t=\infty$ in (8), $4x+m=0$, $x=-\frac{1}{4}m$, the equation of the directrix to the parabola.

CRELLE'S JOURNAL.

NOTE, by CHRISTINE LADD, Chelsea, Mass.—The last number of Crelle's *Journal for pure and applied Mathematics* opens with the continuation of an article by Stern on The Theory of Euler's Numbers.

Sturm contributes a paper of forty pages on Cubic Curves in Space. Six points in space determine such a curve. Through five fixed points, quadratic curves can be passed, through four points, biquadratic curves. The author proposes to determine how many curves of each of these two classes will cut a given line, how many will pass through a given point, how many be tangent to the line, how many cut a plane, how many osculate the plane, and how many will satisfy at once all possible combinations of these conditions. Alsatia shows its mathematical activity by a paper of Milinowski's